Set theory

Set :- A set or a Collection is a group of objects having some well defined and Common properties. The individual object of Collection or set is called an Element or a member of the set.

Ex-

N : for the set of natural Nos.
Z : set of Integers.
C : Set of Complex Nos.

Set Notation - If an element a belongs to set A then we expressed it

$a \in A$

A set Can be specified in following two ways.

(a) **Roster Method (Tabular form) :-** In this method, a set is represented by listing all its elements within braces. { }

Ex- The set of vowel - {a, e, i, o, u}

(b) **Rule Method (Set-builder form) :-** in this Method set is represented by listing all its elements ~~within~~ in terms of one or several characterstics properties.

Ex-

$N = \{x : x$ is a Natural no.$\}$

Some Examples of set Notation by Roster and Rule Method -

(i) Set of all odd No.

(a) $O = \{1, 3, 5 \ldots\}$ (Roster Method)
(b) $O = \{x : x$ is an odd No.$\}$ (Rule Method)

(ii) Set of all odd No. b/w 2 and 10

(a) $B = \{3, 5, 7, 9\}$ (Roster Method)
(b) $B = \{x : x > 0, \; 2 < x < 10\}$ (Rule Method.

Types of set :

① Empty set :- A set is said to be empty or null or void set if it has no Element and it is denoted by ϕ.

Ex - ① A set of Even no. greater than 6 and less than 8.

② Singleton set :- A set Consisting of a Single Element is called a singleton set

Ex - $\{\phi\}$ is a set whose only element is a null set, therefore $\{\phi\}$ is singleton.

③ finite Set :- A set is finite if it consists of a finite no. of different element i.e. if in Counting the different members of the set.

Ex- If A is the set of the week days.

Infinite Set :- A set is infinite if it contains infinite no. of elements i.e. it is not possible to count all members of the set.

Ex- $B = \{2, 4, 6, 8 \ldots\}$

Cardinal numbers of a finite set :- The no. of elements in a finite set A is called cardinal no. or order of the set A and is denoted by $n(A)$

Equivalent Sets :- Two finite set A and B are equivalent if their cardinal nos. are same. i.e.

$$n(A) = n(B)$$

Equal Set :- Two set A and B are said to be equal if every element of A is a member of B and every element of B is a member of ~~A~~ A.

If. $A = \{2, 3, 4, 5\}$ and $B = \{4, 5, 3, 2\}$ then $A = B$, because each element of A is an element of B.

$n(A) = n(B)$

Equal Set :- Two set A and B are said to be Equal if every element of A is a member of B and every element of B is a member of ~~A~~ A.

if. $A = \{2, 3, 4, 5\}$ and $B = \{4, 5, 3, 2\}$
then $A = B$, because each element of A is an element of B.

Subsets :- if every element in a set A is also a member of a set B then A is called a subset of B.
A is subset of B if
$x \in A \Rightarrow x \in B$

Ex- Write the subsets of the following set-
$A = \{1, 2, 3\}$.

Soln- The required subsets are
ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$, $\{1,2,3\}$

Power set :- The set of all subsets of a given set A is called the power set of A and is denoted by the symbol $P(A)$

i.e. $P(A) = \{T : T \subset A\}$
ϕ and A are both member of $P(A)$

Ex. $A = \{a, b\}$
$P(A) = \{\phi, \{a\}, \{b\}, \{a,b\}\}$

Universal Set :- A set that contains all sets in a given context is called the universal set.

Ex- if $A = \{1, 2\}$, $B = \{2, 4, 5\}$ and $C = \{1, 3, 5, 7\}$

then
$U = \{1, 2, 3, 4, 5, 7\}$ Ans.

Basic operation on Sets :-

Union of Sets :- Let A and B be two given sets. The set which contains every element contained in A or B or both A and B is called the union of A and B. Infact union is an 'either' or 'idea'. The symbol $\cup$ is used to denote the union of sets. Thus $A \cup B$ is read as A union B. Symbolically –

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

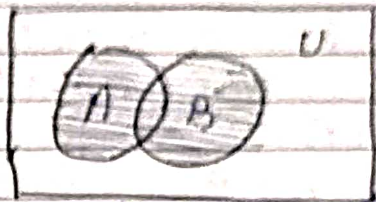
The shaded part represents $A \cup B$ in the following Venn diagram.

Ex –

$A = \{1, 2, 3\}$
$B = \{2, 4, 6\}$

then

$A \cup B = \{1, 2, 3, 4, 6\}$



$A \cup B$ (Shaded)

Properties of union of Sets :–

(i) $A \cup A = A$
(ii) $A \cup B = B \cup A$
(iii) $A \cup \phi = A$
(iv) $A \cup U = U$
(v) If $A = B$ then $A \cup B = A = B$
(vi) If A, B and C are three sets that

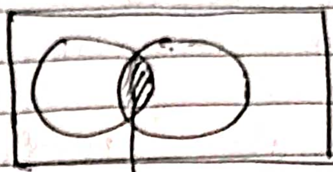
$$A \cup (B \cup C) = (A \cup B) \cup C$$

Intersection of Sets :– Let A and B are two given sets. The set containing all the element which are contained in A as well in B is called the intersection of A and B. The symbol $\cap$ is used to denote the intersection of sets. Thus $A \cap B$ is read as 'the intersection of A and B.

Symbolically $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Ex- if $A = \{a, b, i, z\}$ and $B = \{b, c, i, u, v\}$
then $A \cap B = ?$

$A \cap B = \{i, b\}$



$A \cap B$ (Shaded)

(ii) $A = \{1, 2, 3, 4, 5, 6\}$
$B = \{2, 6, 8, 9\}$
then $A \cap B = ?$
$\therefore A \cap B = \{2, 6\}$

Properties of intersection of sets :-

(i) $A \cap A = A$
(ii) $A \cap B = B \cap A$
(iii) $A \cap B \subset A$ and $A \cap B \subset B$
(iv) $A \cap \phi = \phi$
(v) $A \cap U = A$
(vi) if $A = B$ then $A \cap B = A = B$.
(vii) for disjoint set $A \cap B = \phi$
(viii) $(A \cap B) \cap C = A \cap (B \cap C)$

Disjoint Sets :- Two sets A and B are said to be disjoint sets if they have no element in common. i.e. if their intersection is a Null set i.e.

$A \cap B = \phi \Rightarrow A$ and B are disjoint.

1.5.2 Union of Two Sets

The union of two sets A and B is represented by shaded portion of the venn diagram given below

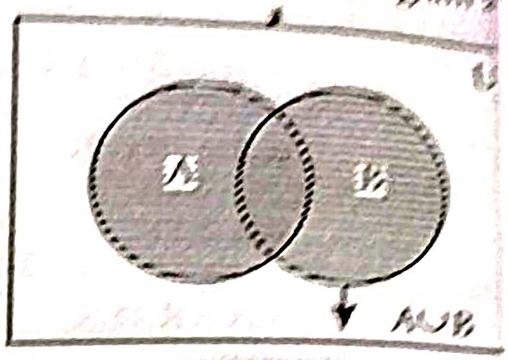


Fig. 1.2

1.5.3 Intersection of Two Sets

The intersection of two sets A and B is shown below by shaded portion

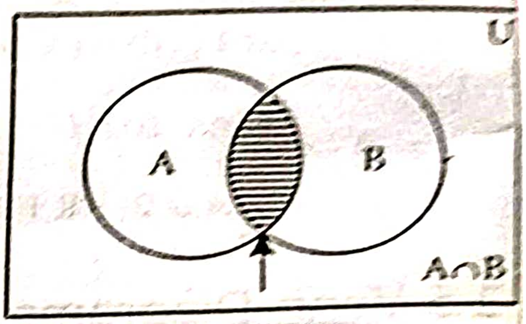


Fig. 1.3

1.5.4 Disjoint Sets

Two disjoint sets are shown by venn diagram as given below

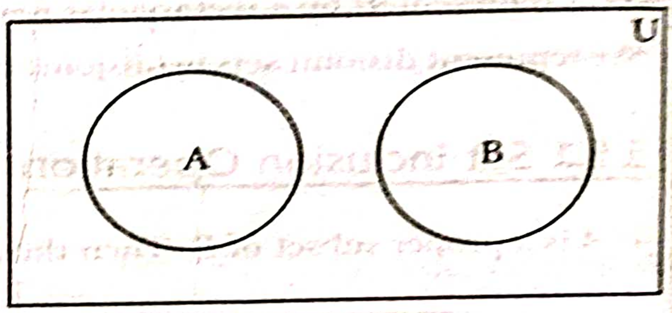


Fig. 1.4

1.5.5 Complement of a Set

The complement of a set A can be represented by shaded portion as given belows

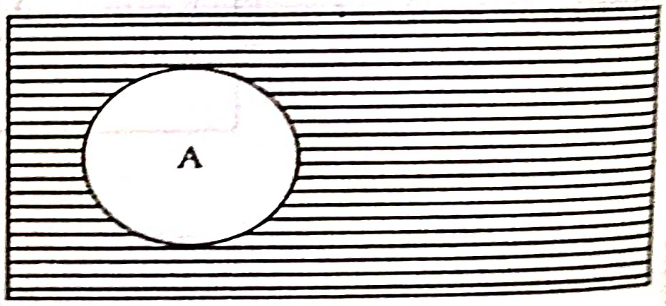


Fig. 1.5

Counting Principle : If A, B and C are finite sets and U be the finite universal set then.

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow$ A, B are disjoint

$n(A \cup B) = \phi$

(iii) $n(A - B) = n(A) - n(A \cap B)$

$n(A - B) + n(A \cap B) = n(A)$

(iv) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Ordered Pair :- If x and y be any two elements then (x, y) is called their orded Pair.

$x \rightarrow$ first member or first Co-ordinate

$y \rightarrow$ Second member or Second Co-ordinate.

$(a, b) = (c, d) \Longleftrightarrow a = c, b = d$

Ex:- if ordered Pair $(2x-1, -5)$ and $(x, y+1)$ are Equal then.

$2x - 1 = x$
$x = 1$

$-5 = y + 1$
$\boxed{y = -6}$

Cartesian Product :- If A and B are any two sets, then set of all distinct ordered pairs whose first coordinate is an element of A and whose 2nd co-ordinate is an element of B is called the Cartesian Product of A and B is denoted by A×B.

$A \times B = \{(a, b) ; a \in A \text{ and } b \in B\}$

Ex-

$A = \{a, b, c\}, \quad B = \{x, y, z\}$

$A \times B = \{(a, x)\ (a, y)\ (a, z), (b, x), (b, y), (b, z), (c, x), (c, y), (c, z)\}$

$B \times A = \{(x, a)\ (x, b)\ (x, c), (y, a)\ (y, b)\ (y, c), (z, a)\ (z, b)\ (z, c)\}$

Note - ① $A \times B \neq B \times A$ i.e. Cartesian Product is commutative.

If the set A and B have m and n elements respectively, then the set

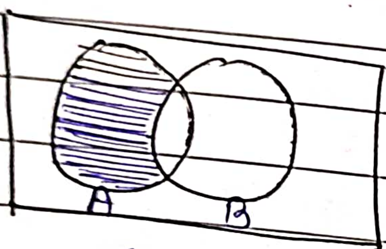
if either A or B is a Null set then the set A×B is also a Null set.

$A \times B = B \times A \Rightarrow A = B$.

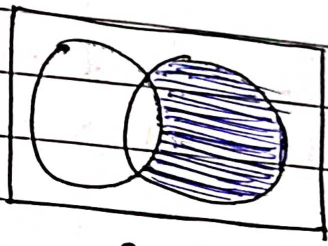
Ex- if $A = \{a, b, c\}$ and $B = \{p, q, r\}$
then $A \cap B = \phi$

Difference of two set :- Let A and B be two set. the difference of A and B. written as $A - B$ is the set of all those element of A which do not belong to B.
thus

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$



A − B (shaded)



B − A (shaded)

Ex- $A = \{2, 3, 4, 5, 6, 7\}$
$B = \{3, 5, 7, 9, 11, 13\}$
then $A - B = ?$
$A - B = \{2, 4, 6\}$
$B - A = \{9, 11, 13\}$

Complement of a Set :- the Complement of a given set A is defined as the set Consisting of those elements of the universal set which are not Contained in the given set A.
It is denoted by A' or $\bar{A}$.
Symbolically,

$$A' = \{x : x \in U, x \notin A\}$$

25

Fig. 1.11

Example 24: A school has 21 boys in basket ball team, 26 in hockey and 29 in football team. Now if 14 boys play hockey and basketball, 15 boys play hockey and football, 12 boys play football and basketball and 8 boys play hockey, football and basketball all three games, then what is total number of boys playing these games?

[B.C.A. (Rohilkhand) 2007]

Solution: Let B, H, F denote the total number of boys playing basketball, hockey and football.

Then $$n(B) = 21, \quad n(H) = 26, \quad n(F) = 29$$

$$n(H \cap B) = 14; \quad n(H \cap F) = 15; \quad n(F \cap B) = 12$$

and $$n(B \cap H \cap F) = 8$$

Then $$n(B \cup H \cup F) = n(B) + n(H) + n(F) - n(H \cap B) - n(H \cap F) - n(F \cap B) + n(B \cap H \cap F)$$

$$= 21 + 26 + 29 - 14 - 15 - 12 + 8$$

$$= 84 - 41$$

$$= 43$$

Example 26: A survey shows that 73% of the Indians like apples, whereas 65% like oranges. What percentage of Indians like both apples and oranges? **[B.C.A. (R) Meerut 2003]**

Solution:

Let A = Set of Indians who like apples

B = Set of Indians who like oranges

Then $n(A) = 73, \quad n(B) = 65, \quad n(A \cup B) = 100$

$\therefore$ $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 73 + 65 - 100 = 38$

Hence, 38 of the Indians like both apples and oranges.

Example 28: *In a group of 52 persons, 16 drink tea but not coffee and 33 drink tea.*

(i) *How many drink tea and coffee both?*

(ii) *How many drinks tea and coffee both?*

(iii) *How many drink coffee but not tea?*

[B.C.A. (Delhi) 2002, 2

Solution:

Let A = Set of persons who drink tea

and B = Set of persons who drink coffee

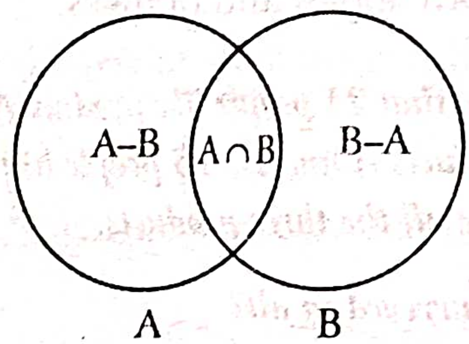


Fig. 1.14

Then, $A-B$ = Set of persons who drink tea but not coffee

And $B-A$ = Set of persons who drink coffee but not tea

But $n(A\cup B)=52,\ n(A-B)=16$ and $n(A)=33$

(i) Set of persons who drink tea and coffee both $=(A\cap B)$

Now, $n(A-B)+n(A\cap B)=n(A)$

$\Rightarrow$ $n(A\cap B)=n(A)-n(A-B)=(33-16)=17$

Thus, 17 drinks tea and coffee both

(ii) $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$\Rightarrow$ $n(B)=n(A\cup B)+n(A\cap B)-n(A)=52+17-33=36$

Now $n(B-A)+n(A\cap B)=n(B)$

$\Rightarrow$ $n(B-A)=n(B)-n(A\cap B)=36-17=19$

Thus, 19 drinks coffee but not tea.

Example 29: *In a group of 850 persons, 600 can speak Hindi and 340 can speak Tamil.*

Find:

(*i*) *How many can speak both Hindi and Tamil?*

(*ii*) *How many can speak Hindi only?*

(*iii*) *How many can speak Tamil only?*

[B.C.A. (Meerut) 20

Solution: Let A = Set of persons who can speak Hindi.

B = Set of persons who can speak Tamil

$\therefore \quad n(A) = 600, \quad n(B) = 340 \quad \text{and} \quad n(A \cup B) = 850$

(i) Set of persons who can speak both Hindi and Tamil $= (A \cap B)$

$\therefore \quad n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$$= 600 + 340 - 850 = 90$$

Thus, 90 persons can speak both Hindi and Tamil

(ii) Set of persons who can speak Hindi only $= (A - B)$

Now $\quad n(A - B) + n(A \cap B) = n(A)$

$\Rightarrow \quad n(A - B) = n(A) - n(A \cap B) = 600 - 90 = 510$

Thus, 510 persons can speak Hindi only

(iii) Set of persons who can speak Tamil only $= (B - A)$

Now $\quad n(B - A) + n(A \cap B) = n(B)$

$\Rightarrow \quad n(B - A) = n(B) - n(A \cap B) = 340 - 90 = 250$

Hence, 250 persons can speak Tamil only.

...English and 48% failed in Science.

Example 36: *If* $A = \{1, 2, 3\}, B = \{4, 5\}$ *then show that* $A \times B \neq B \times A$.

Solution: $A \times B = \{1, 2, 3\} \times \{4, 5\}$

$$= \{(1,4), (1,5), (2,4), (2,5), (3,4), (3 \times 5)\}$$

$$B \times A = \{4, 5\} \times \{1, 2, 3\}$$

$$= \{(4,1), (4,2), (4,3), (5,1), (5,2), (5,3)\}$$

Hence $\quad A \times B \neq B \times A$

Example 37: *If* $A = \{a, b\}, B = \{2, 3, 5, 6, 7\}$ *and* $C = \{5, 6, 7, 8, 9\}$ *then* $A \times (B \cap C)$

[B.C.A. (Lucknow) 2010]

Solution: $B \cap C = \{2, 3, 5, 6, 7\} \cap \{5, 6, 7, 8, 9\}$

$$= \{5, 6, 7\}$$

$$A \times (B \cap C) = \{a, b\} \times \{5, 6, 7\}$$

$$= \{(a,5), (a,6), (a,7), (b,5), (b,6), (b,7)\}$$

Example 38: *If* $A = \{1, 2, 3\}, B = \{2, 3, 4\}, C = \{1, 3, 4\}$ *and* $D = \{2, 4, 5\}$ *than verify that* $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

Solution: $A \times B = \{1, 2, 3\} \times \{2, 3, 4\}$

$$= \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$$

$$C \times D = \{1, 3, 4\} \times \{2, 4, 5\}$$

$$= \{(1,2), (1,4), (1,5), (3,2), (3,4), (3,5), (4,2), (4,4), (4,5)\}$$

$\therefore$
$$(A \times B) \cap (C \times D) = \{1,2), (1,4), (3,2), (3,4)\}$$

Also
$$A \cap C = \{1, 3\} \qquad \text{and} \qquad B \cap D = \{2, 4\}$$

$\therefore$
$$(A \cap C) \times (B \cap D) = \{1,2), (1,4), (3,2), (3,4)\}$$

Hence
$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

(VI) De Morgan's laws :- If A and B are any two ~~any~~ sets.

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Proof- (i) Let x be an arbitrary element of $(A \cup B)'$ then.

$x \in (A \cup B)'$

$x \notin (A \cup B)$ or

$\Rightarrow$ $x \notin A$ and $x \notin B$

$x \in A'$ and $x \in B'$

$x \in A' \cap B'$

$(A \cup B)' \subset A' \cap B'$

Again,

Let y an arbitrary element of $A' \cap B'$ then,

$y \in A' \cap B'$

$\Rightarrow$ $y \in A'$ and $y \in B'$

$y \notin A$ or ~~and~~ $y \notin B$

$\Rightarrow$ $y \notin A \cup B$

$\Rightarrow$ $y \in (A \cup B)'$

$\therefore$

$A' \cap B' \subset (A \cup B)'$

Hence

$(A \cup B)' = A' \cap B'$

ii) Let x be an arbitrary element of $(A \cap B)'$ th

$x \in (A \cap B)'$

$\Rightarrow$ $x \notin (A \cap B)$

$x \notin A$ and $x \notin B$

$x \in A'$ or $x \in B'$

$x \in A' \cup B'$

$(A \cap B)' \subseteq A' \cup B'$

Again let y be an arbitrary element of $A' \cup B'$ then

$y \in (A' \cup B')$

$y \in A'$ or $y \in B'$

$y \notin A$ and $y \notin B$

$y \notin (A \cap B)$

$y \in (A \cap B)'$

$\therefore$ $A' \cup B' \subseteq (A \cap B)'$

Hence

$(A \cap B)' = A' \cup B'$